ENTE PER LE NUOVE TECNOLOGIE L'ENERGIA E L'AMBIENTE

Stability of a distributed generation network using the Kuramoto models

Vincenzo Fioriti[1], Silvia Ruzzante[2], Elisa Castorini[1], Elena Marchei[2], and Vittorio Rosato[1]

[1]ENEA, Centro Ricerche Casaccia, Roma
[2]ENEA, Centro Ricerche Portici, Napoli

CRITIS '08
FRASCATI 13 ottobre 2008

Summary

- In the near future, a major source of concern for the continuity of service of the Power System will be the Distributed Generation (DG).

  A Distributed Generation system is a set of small, heterogeneous, independent, interconnected power plants phase-synchronized to oscillate at the same frequency; otherwise a blackout may occur.

- In order to study this problem we analyze the phase stability of a small size distributed generation grid (the network), by means of a modified Kuramoto Model.

- In our model, contrarily to the standard Kuramoto model (SKM), the strength of the couplings is randomly chosen and allowed to vary; couplings are considered as power from generators (following Filatrella 2008) .

- Although the network undergoes several synchronization losses, it is able to quickly resynchronize. Useful hints for distributed generation grid design are given.

  Moreover, we note that the word synchronization should be intended as a synonym for inter-dependency.

Some examples

Oscillators able to synchronize their rhythmic features:

Circadian rhythms
Many living organisms synchronize to the day-night cycle.

Electrical generators
All of the generators producing power on a power grid must be synchronized to one another.

Josephson junction arrays
Josephson junctions form systems of oscillators that have been found theoretically to exhibit synchronization.

Heart, intestinal muscles, neurons
The muscles in your heart, for example, must all be synchronized to create a coherent beat.

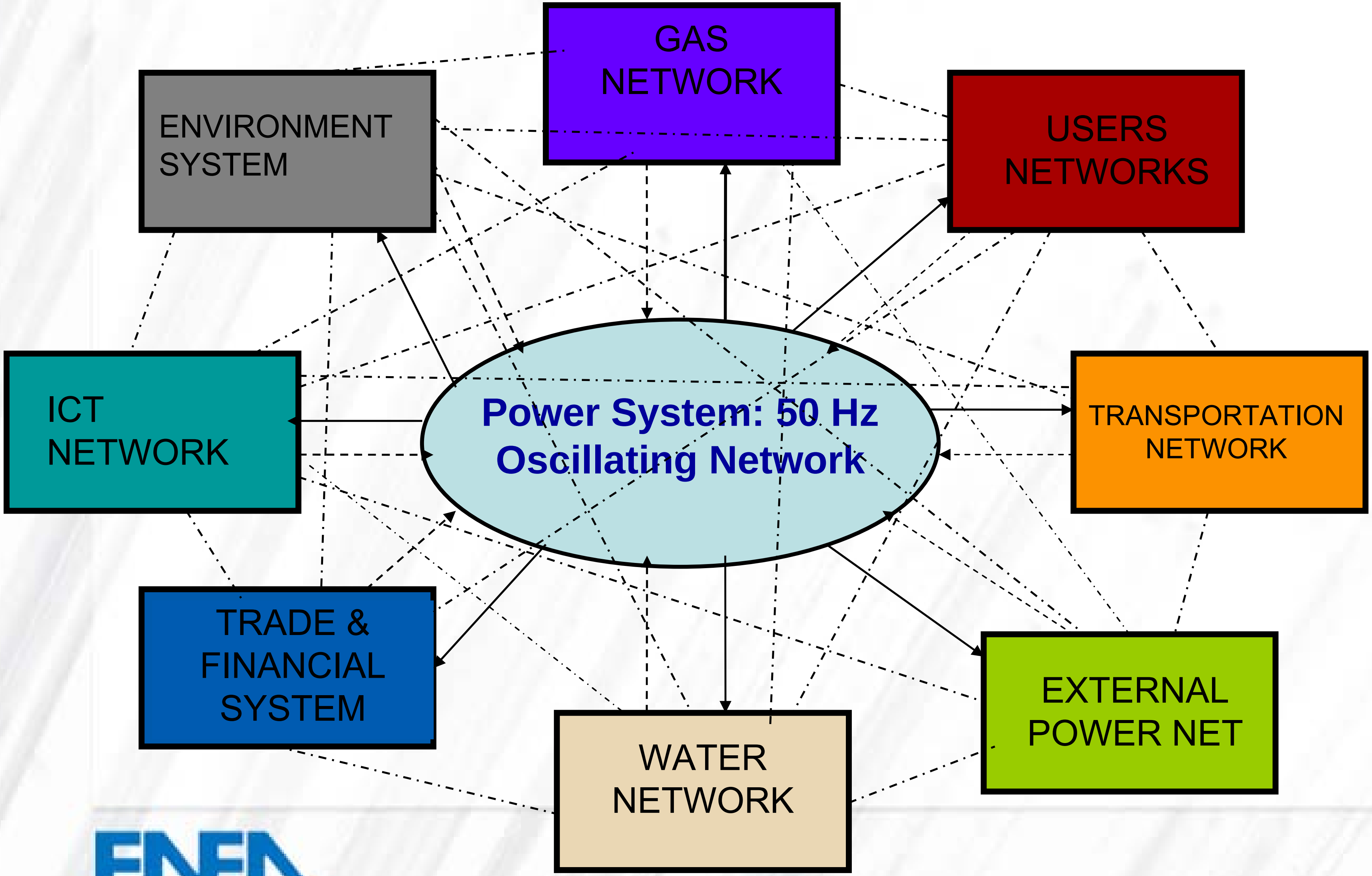
Fireflies
Certain species of fireflies have been found to synchronize, turning on and off at the same time.

Walking pace

Metronomes

Stock market boom / crash

Motivation and Relevance



Motivation and Relevance: Sync & Dependence

CRITIS08

Node *i* depends from node *h*. Generally, node i is syncrhonized with node *h* if:

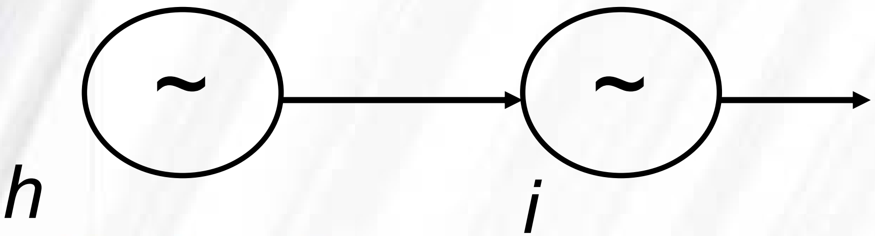
$$\Theta(F_i) = G(\Theta(F_h)) + const$$

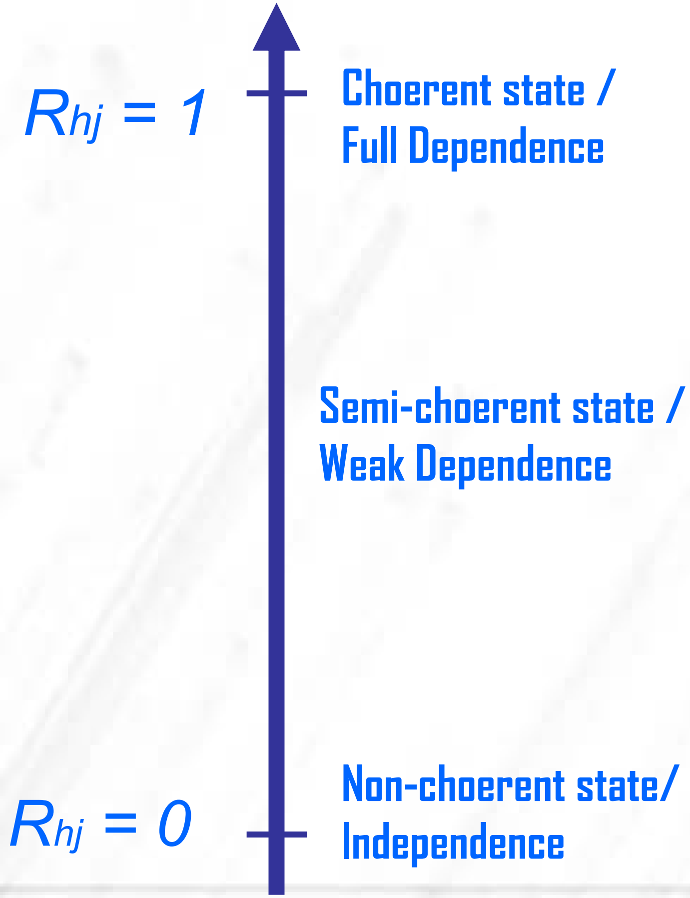
or

$$|F_i| = G(|F_h|) + const$$

Special case: *G* is the identity:

$$\lim_{t \to \infty} |\theta_i(t) - \theta_h(t)| = const$$





ENEA

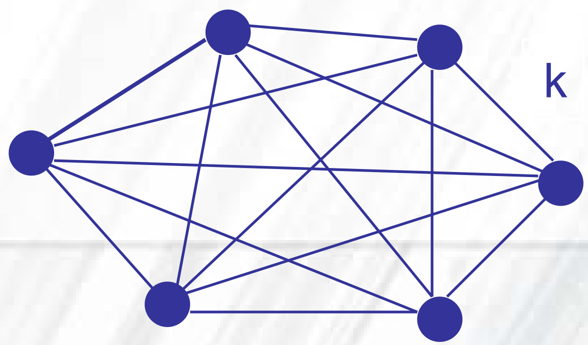
The Kuramoto Equation (SKM)

CRITIS08

$$\dot{\theta}_i = \omega_i + \frac{K}{N}\sum_{j=1}^{N}\sin(\theta_j - \theta_i) \quad i = 1,\ldots,N$$

Kuramoto showed that for *K* < *kc* oscillators remain unsynchronized in phase, while for *K* > *kc* they synchronize .

- $\lim |\theta_i(t) - \theta_j(t)| = const, \quad t \to \infty$
- *N* is the number of oscillators (the nodes)
- ω_i is the nominal frequency of the oscillator i
- *K* is the coupling strength, a fixed and constant scalar in the SKM
- all-to-all couplings



The network

ENEA

The Kuramoto Equation (SKM)

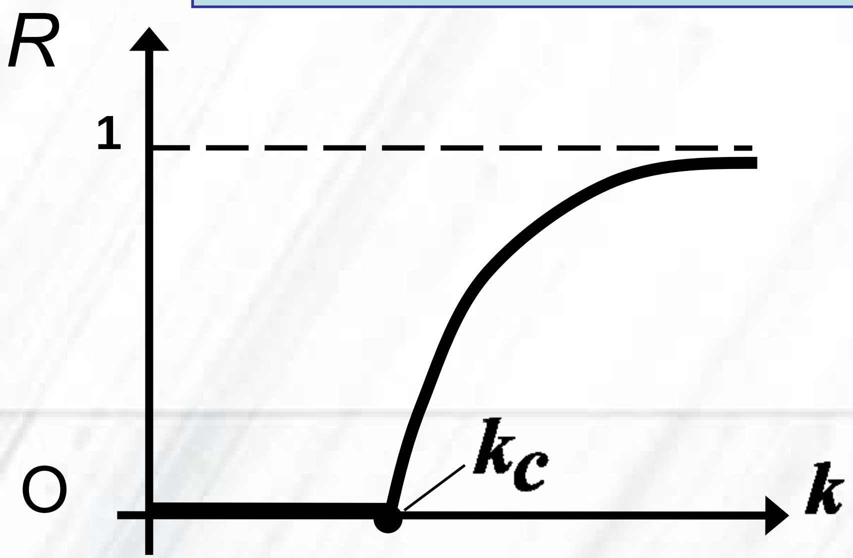
- R is the "order parameter"; ranges between *0* and *1* It can be regarderd as a measure of the quality of synchronization
- Considering two nodes *i* and *h* R_{ih} can be a measure of their inter-dependence
- k_c is the critical coupling value

$$R = \left| \frac{K}{N} \sum_{i=1}^{N} e^{j\theta_i} \right|$$

The order parameter

(here j is the imaginary number)

The bifourcation diagram



The Modified Kuramoto Equation (MKM)

CRITIS08

We modify the SKM introducing an adjacency matrix ***K*** connecting nodes. Its randomly chosen entry K_{ij} represents the ***variable*** coupling strength between nodes as a function of time (4):

$$K = K_m \begin{pmatrix} K_{11} & \cdots & K_{1n} \\ \vdots & \ddots & \vdots \\ K_{n1} & \cdots & K_{nn} \end{pmatrix}$$

$$\dot{\theta}_i = \omega_i + K_m \sum_{j=1}^{N} K_{ij} \sin(\theta_j - \theta_i) \qquad i = 1, \ldots, N \qquad (4)$$

$K = (P_{max}\, \Omega) / \mathrm{I}$

(Filatrella, Eur. Phys. J. B , 2008)

ENEA

Modified Kuramoto Model: the Grid Topology

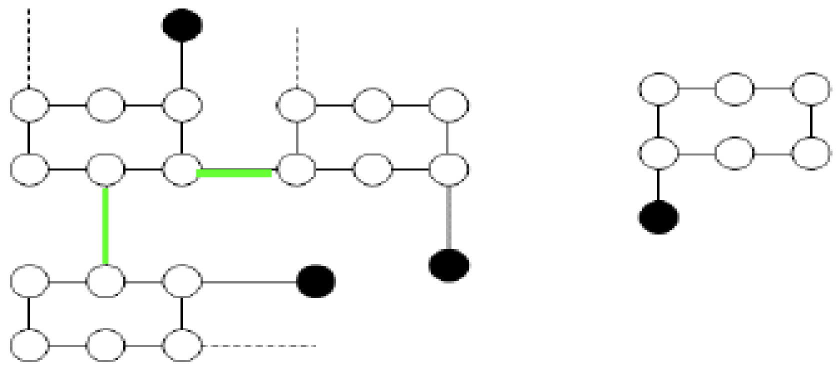
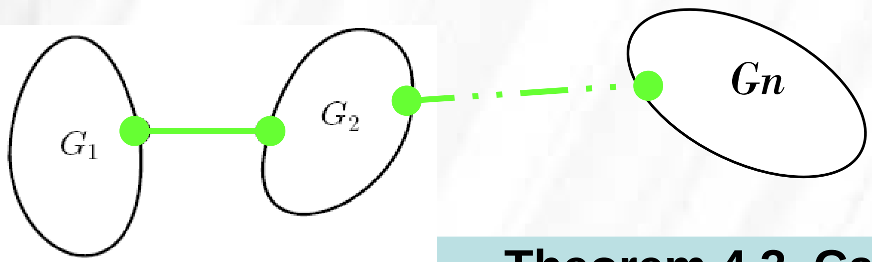


Fig. 1 On the left side the block topology of the oscillators /generators, with the dissipating node. On the right side, a single block. This is the network used for the simulations whose results are the object of the present work.



Theorem 4.3, Canale and Monzon, 2007

If networks *G_1* and *G_2* are "almost" synchronized, also *the resulting network* *$G = \{ G_1 \cup G_2 \ldots \cup G_n \}$* is "almost" synchronized.

Results: low couplings

θ

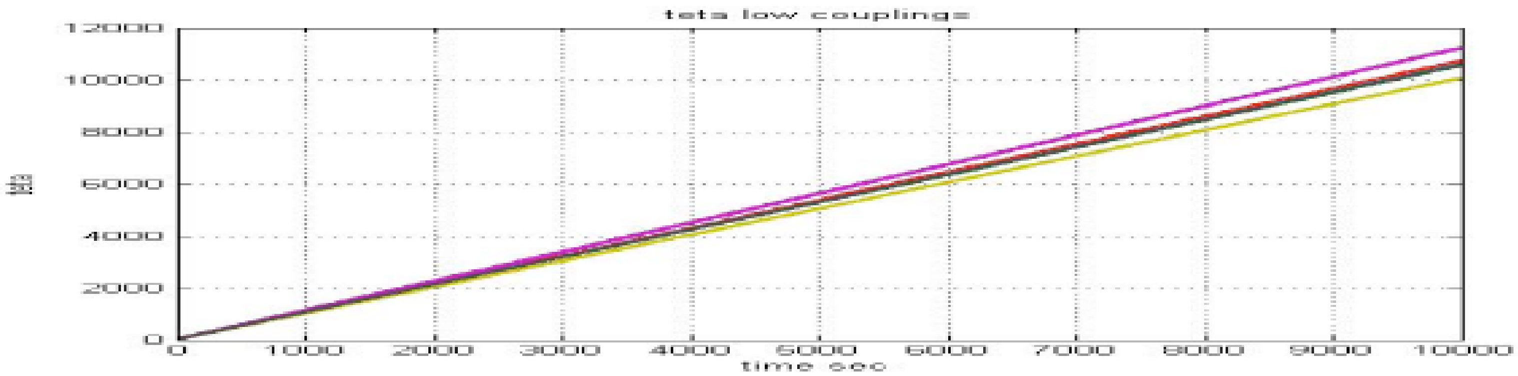


Fig. 2 Phases with low coupling ($K_m = 0.1$).

t

R

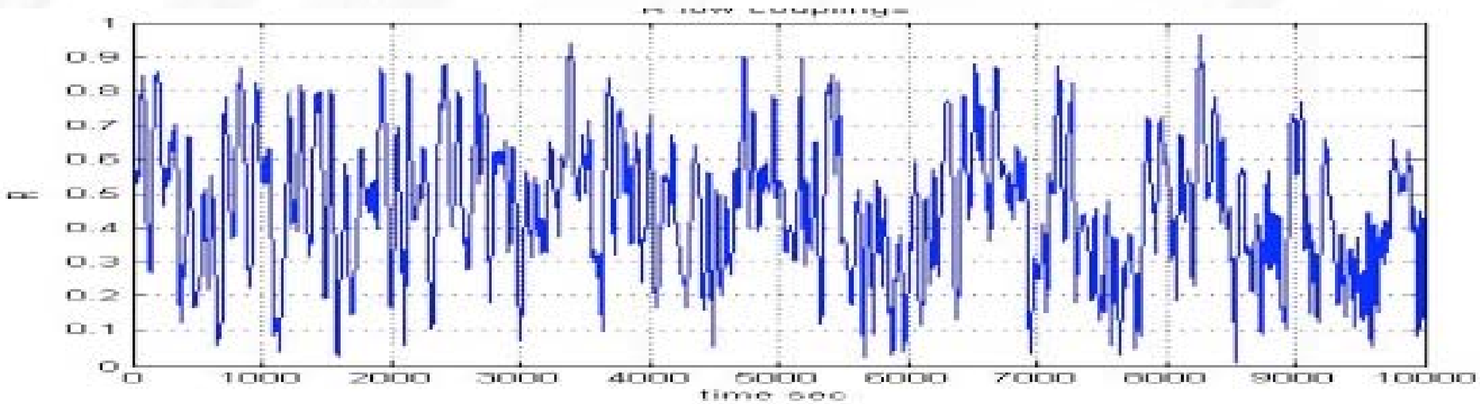


Fig. 3 Order parameter R with low coupling ($K_m = 0.1$).

t

Results: high couplings

θ

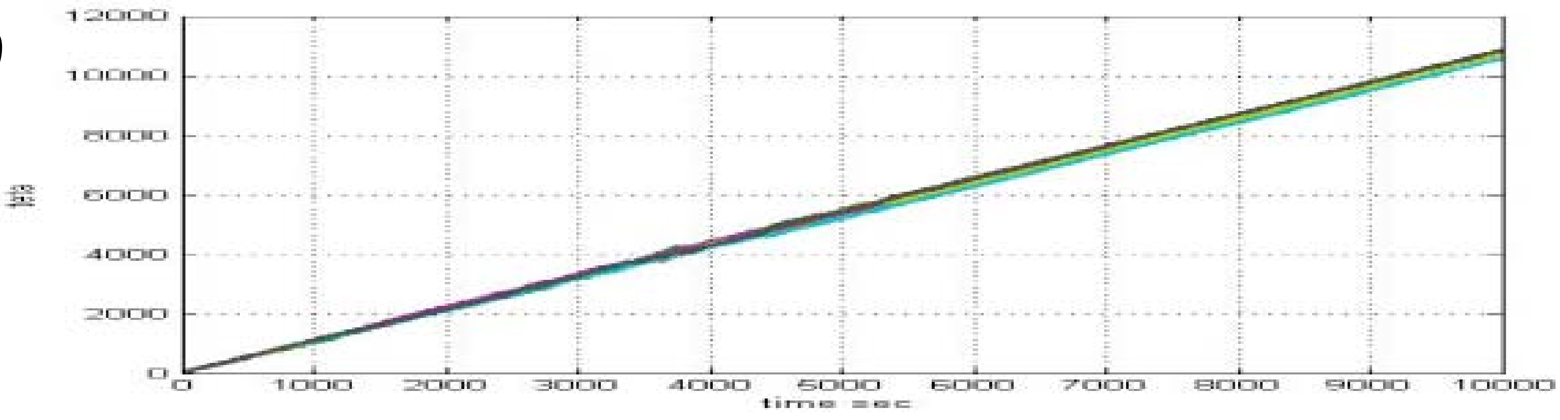


Fig. 4 Phases with high coupling ($K_m = 400$).

t

R

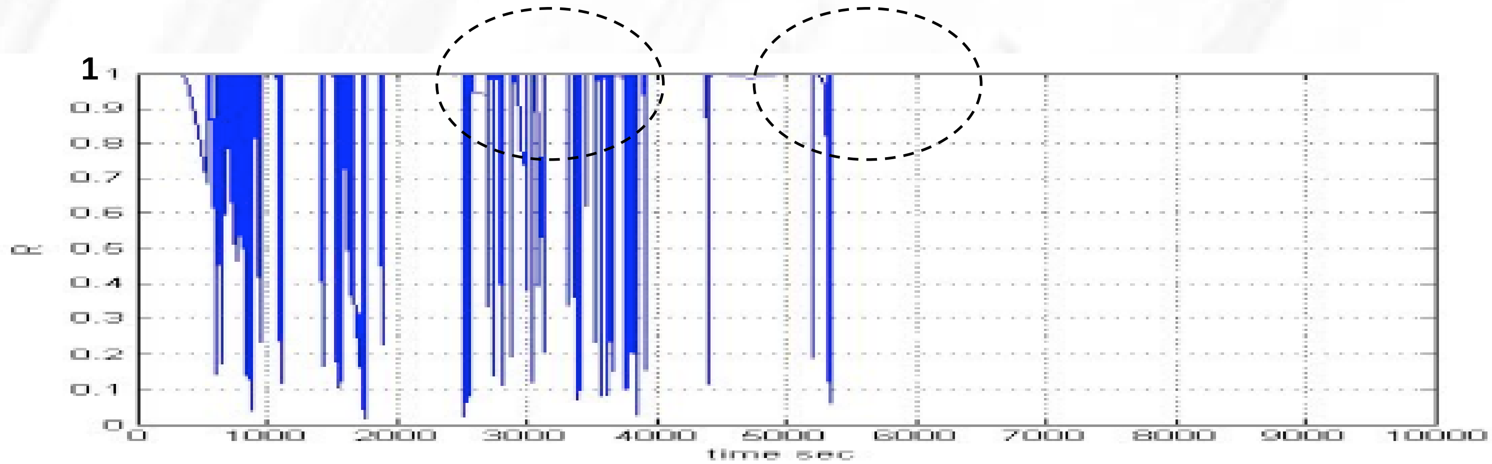


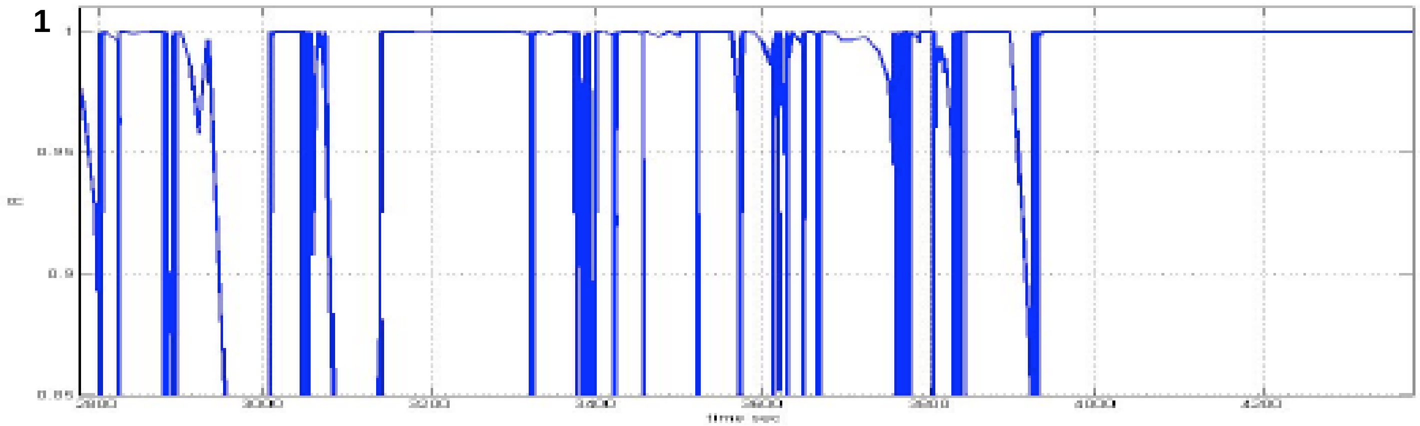
Fig. 5 Order parameter R with high coupling ($K_m = 400$).

t

Results: high couplings

R

1



t

Fig. 7 Enlargement of Fig. 5, (between 3500 and 4000 *s*).

R

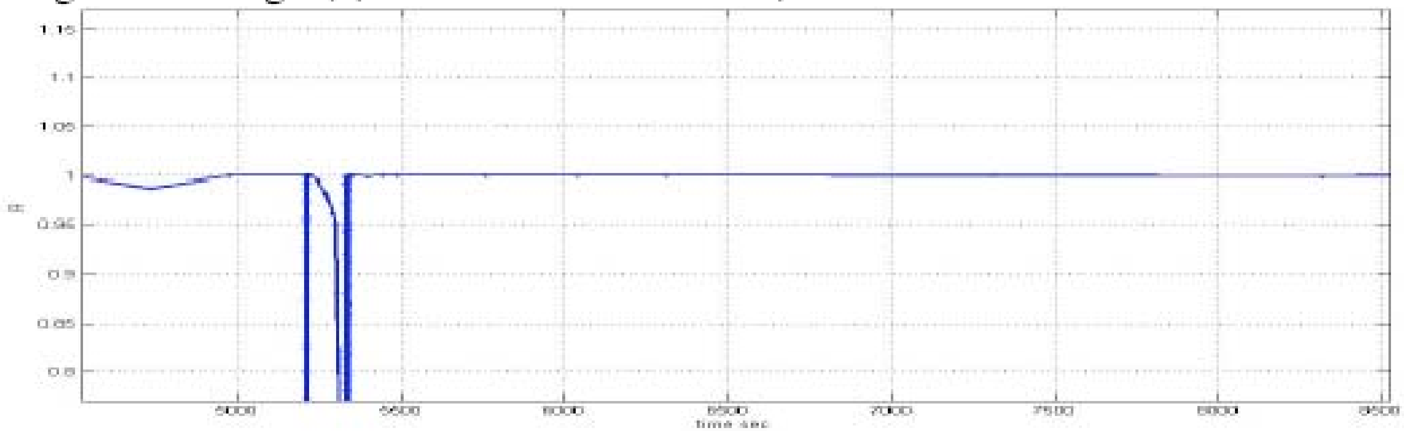


Fig. 6 Enlargement of Fig. 5, $t > 5000\,s$, high coupling.

t

Results: high couplings

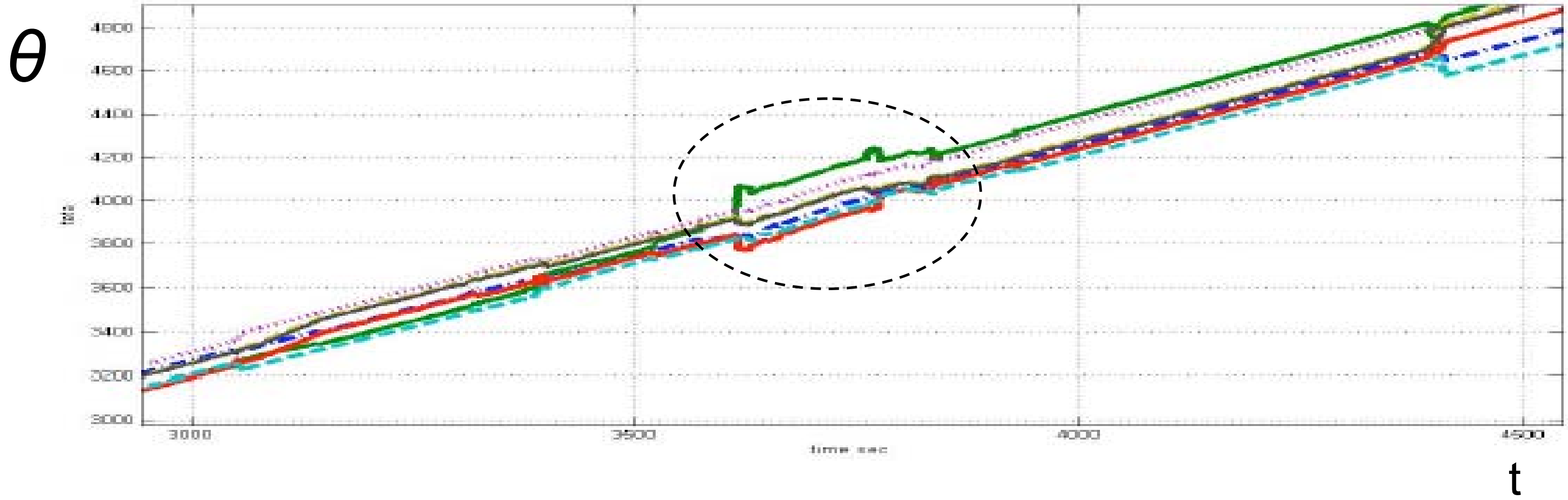
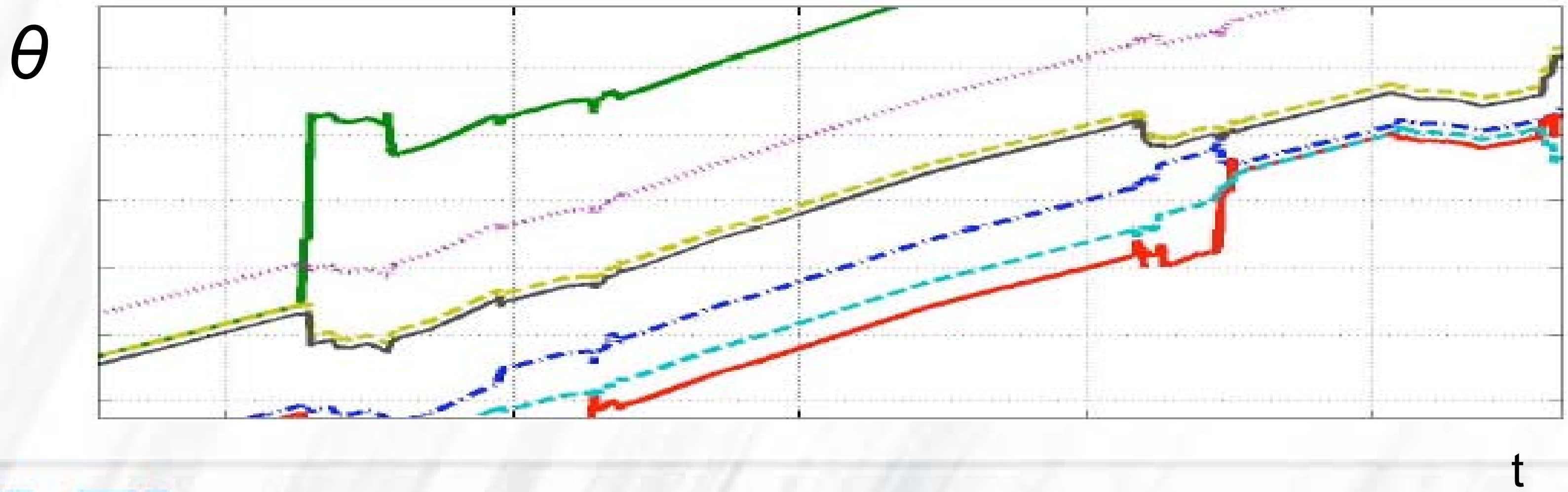


Fig. 8 Phases synchronization crisis (enlargement between 3500 and 4000 s) for $K_m = 400$.



Rising-time

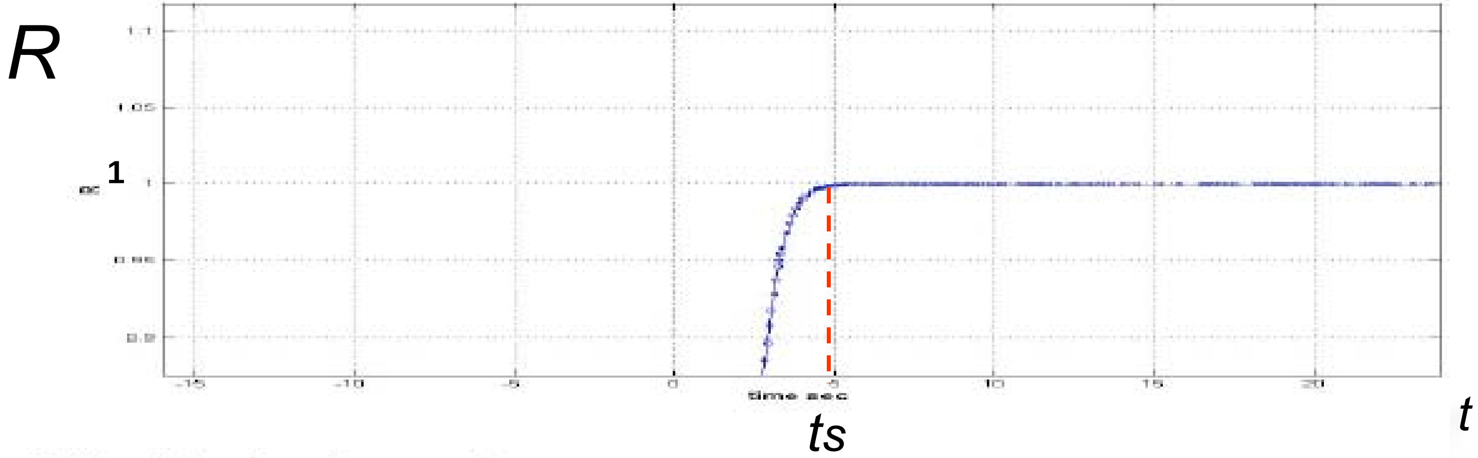


Fig. 10 The rising time, for $K_m = 4$.

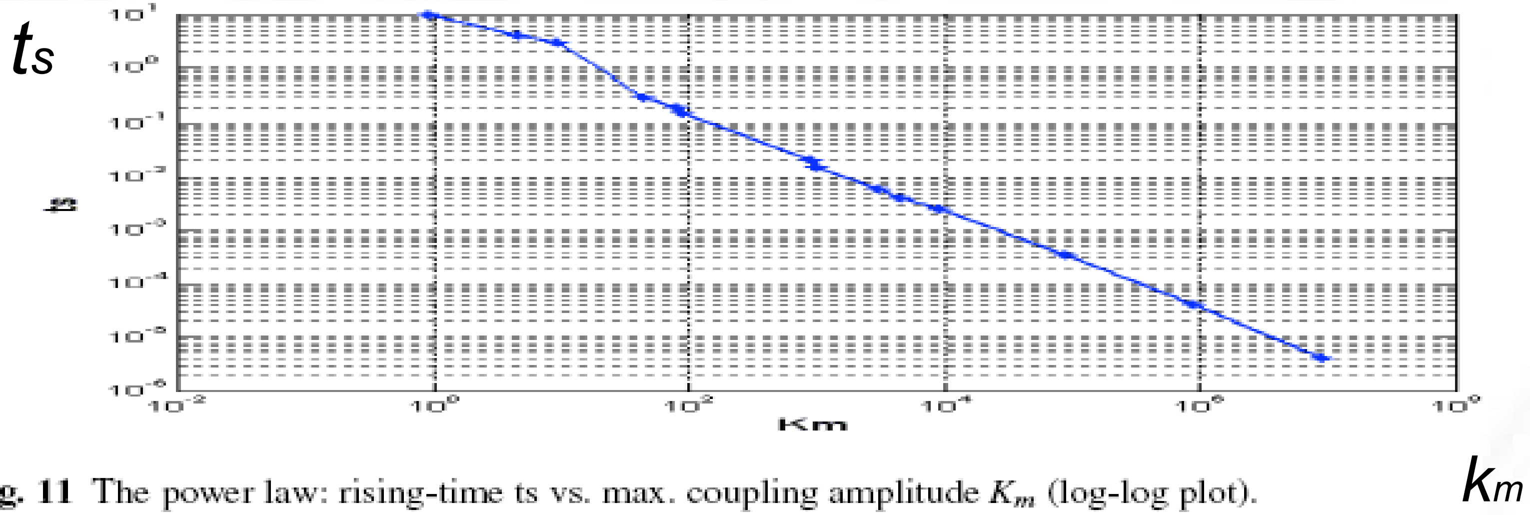
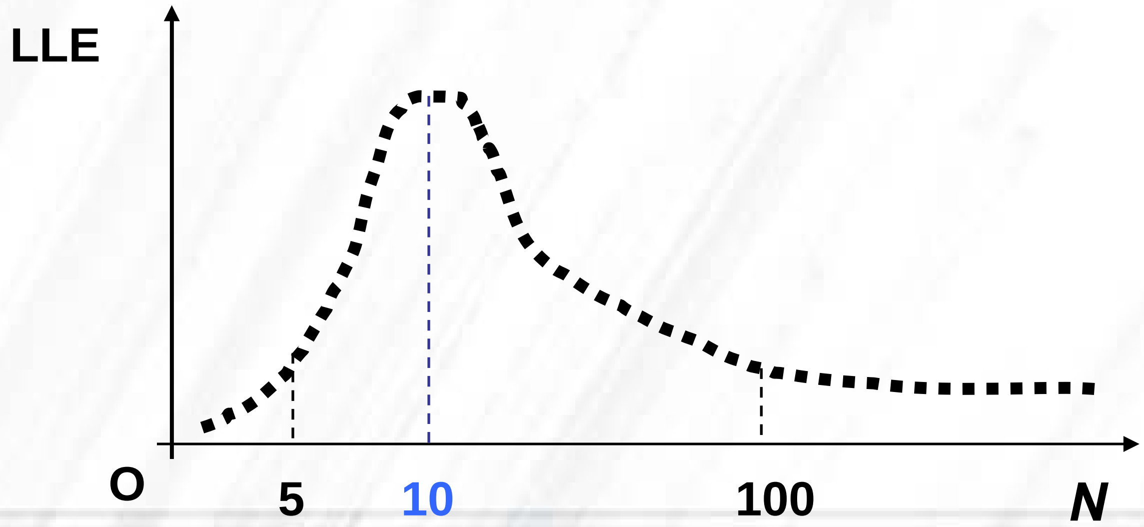


Fig. 11 The power law: rising-time ts vs. max. coupling amplitude K_m (log-log plot).

Network set size

The strongest phase chaos (low coupling case) occurs when the number of nodes in the network is about 10 (Popovych, Phy. Rev. E, 2005).



Order parameter: randomness, chaos and regularity

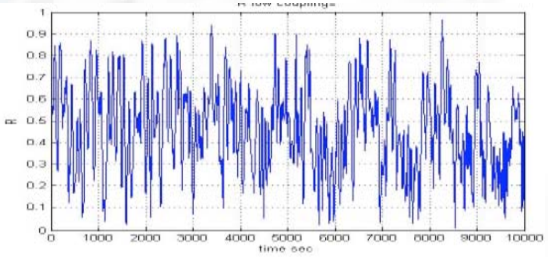
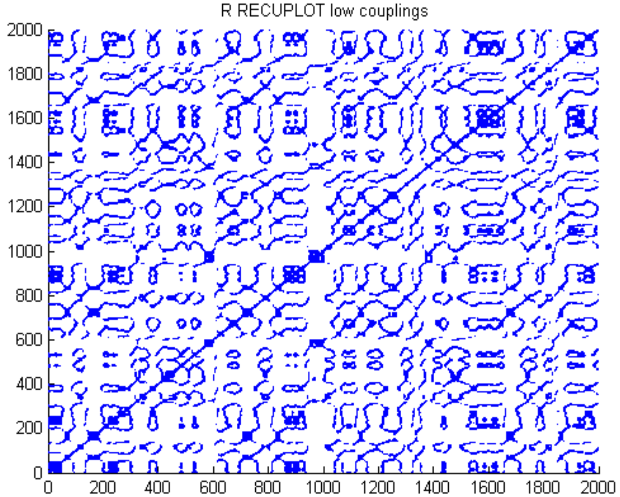
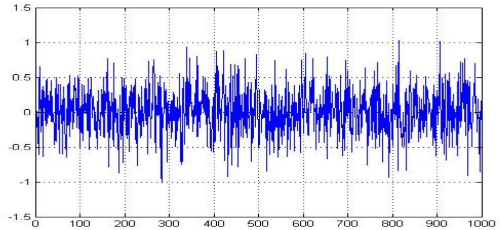


Fig. 3 Order parameter R with low coupling ($K_m = 0.1$).

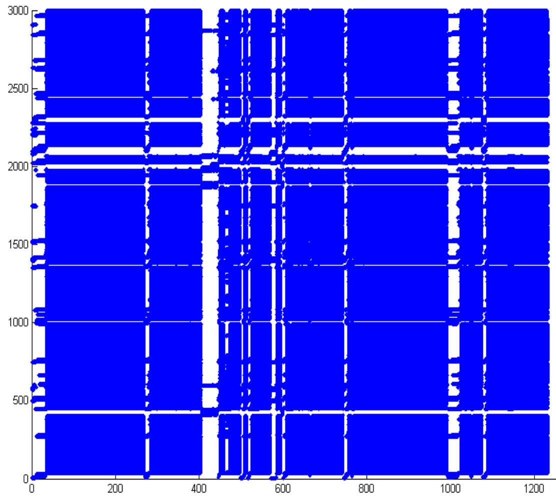
b



b



c



a

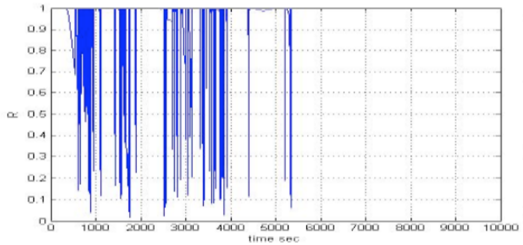
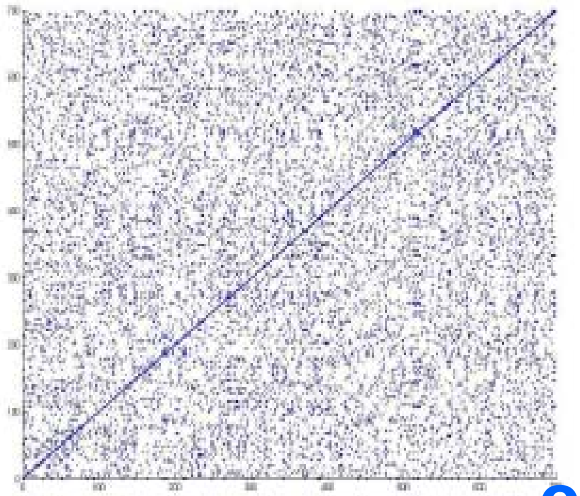


Fig. 5 Order parameter R with high coupling ($K_m = 400$).

a



c

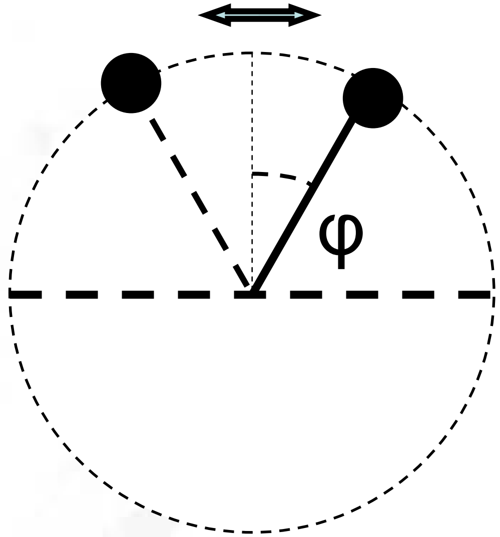
Conclusions

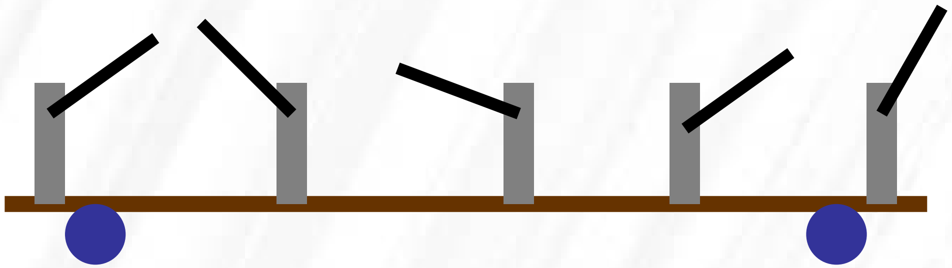
- **The Kuramoto modified model is an appropriate tool to numerically study the DG stability**
- **Voltage of transmission lines should be kept high**
- **The DG grid seems robust with respect to significant perturbations**
- **The order parameter is able to describe the QoS (and the inter-dependencies)**
- **DG grid size must be very small ($N < 5$) or large ($N > 20$)**
- **Proper topology of DG microgrids must be attentively considered**

Future researches:

- **use the MKM to study much larger and realistic DG grids**
- **evaluate the impact of grid topology on the phase stability**

Metronomes





THANK YOU

Cardell-Ilic distributed generation

$$\dot{\mathbf{x}} = \mathbf{A}\mathbf{x} - \tilde{\mathbf{K}}\omega + \dot{\mathbf{p}}$$

$$\dot{x}_i = \sum_j a_{ij}x_j - \sum_j \tilde{k}_{ij}\omega_j + \dot{p}_i \quad i = 1, \ldots, N$$

Only for the state variables regarding the phase. Setting:

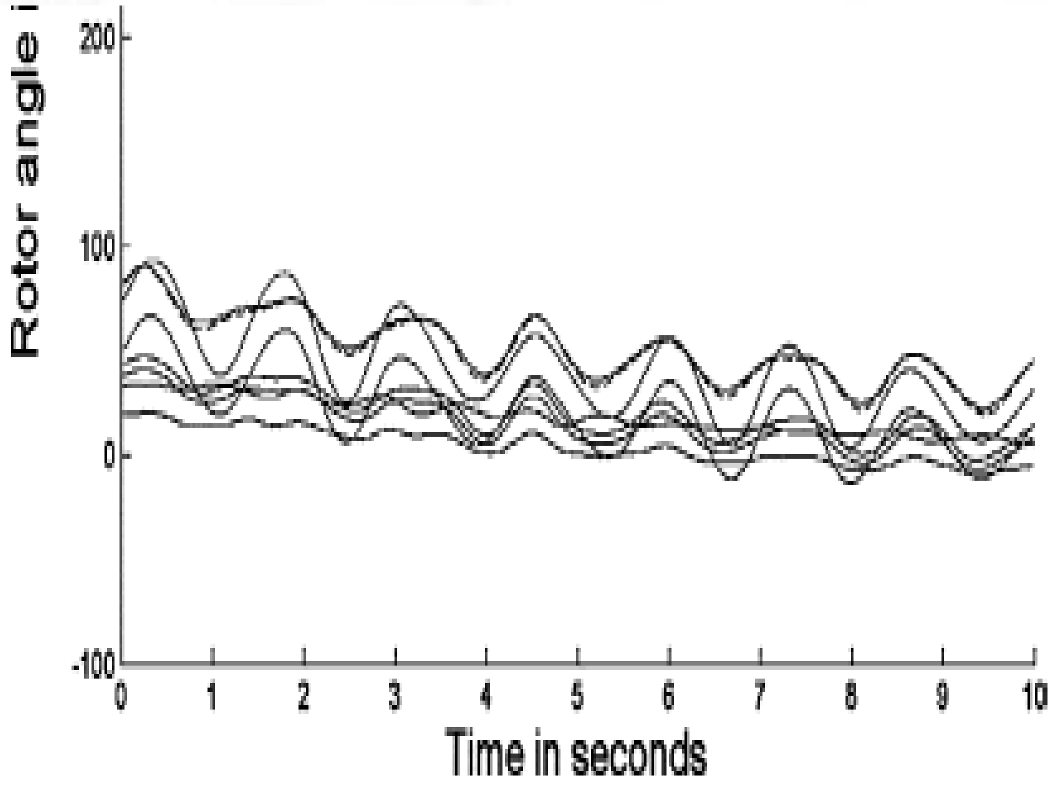
$$\dot{\theta}_i = x_i$$

$$\dot{p}_i = -\frac{\Omega_0}{I_i} p_i^{max} \sum_j \sin(\theta_j - \theta_i) \quad i = 1, \ldots, N$$

where I_i is the inertial moment, Ω_0 the nominal system frequency and p_i^{max} is the maximum power of the i^{th} generator. We derive that:

$$\sum_j a_{ij}\dot{\theta}_j = \sum_j \tilde{k}_{ij}\omega_j + \frac{\Omega_0}{I_i} p_i^{max} \sum_j \sin(\theta_j - \theta_i) \qquad i = 1, \ldots, N$$

Synchrony



AN EXAMPLE

5000 Lorenz systems:

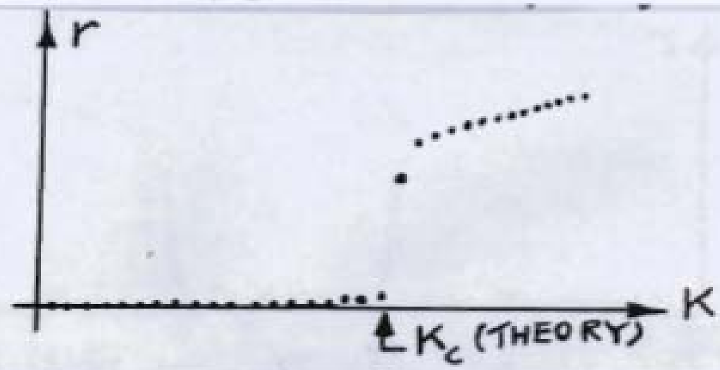
$$\dot{x}_i = 10(y_i - z_i) - k\sum_j A_{ij} x_i$$

$$\dot{y}_i = \rho_i x_i - y_i - x_i z_i$$

$$\dot{z}_i = -\frac{8}{3} z_i + x_i y_i$$

ρ_i chosen from [28,51] : mostly chaotic

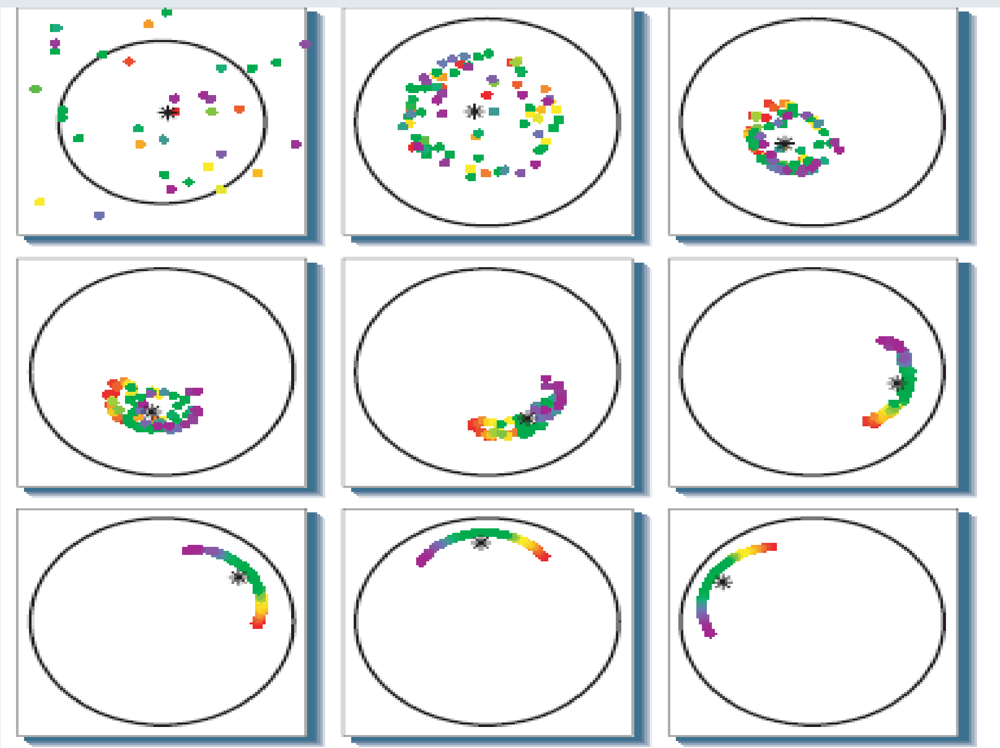
Network: N=5000; $\rho(d) \propto d^{-2.5}, \quad d \geq 100$



References: **Restrepo, Ott, Hunt, Physica D (2006)**

[Ott, So, Barreto, Antonsen, Physica D(2002)]

Strogatz 2001



Steven H. Strogatz

NATURE | VOL 410 | 8 MARCH 2001 |

Figure 2 Spontaneous synchronization in a network of limit-cycle oscillators with distributed natural frequencies. The state of each oscillator is represented geometrically as a dot in the complex plane. The amplitude and phase of the oscillation correspond to the radius and angle of the dot in polar coordinates. Colours code the oscillators' natural frequencies, running from slowest (red) to fastest (violet). In the absence of coupling, each oscillator would settle onto its limit cycle (circle) and rotate at its natural frequency. However, here all the oscillators are also pulled towards the mean field that they generate collectively (shown as an asterisk at the centre of the population). Time increases from left to right, and from top to bottom. Starting from a random initial condition, the oscillators self-organize by collapsing their amplitudes; then they sort their phases so that the fastest oscillators are in the lead. Ultimately they all rotate as a synchronized pack, with locked amplitudes and phases. The governing equations describe a mean-field model of a laser array[23]. (Simulation provided by R. Oliva.)

Order parameter: randomness, chaos and regularity

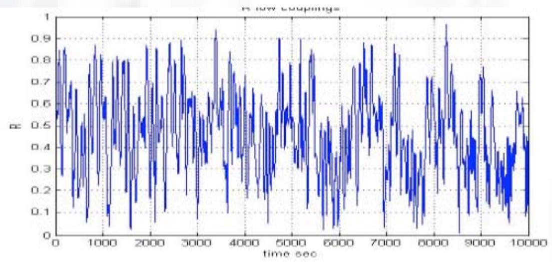
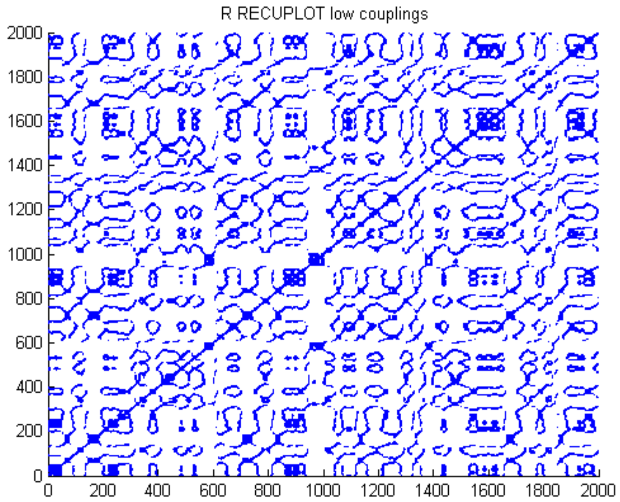
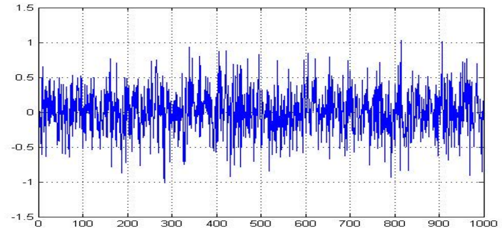


Fig. 3 Order parameter R with low coupling ($K_m = 0.1$).

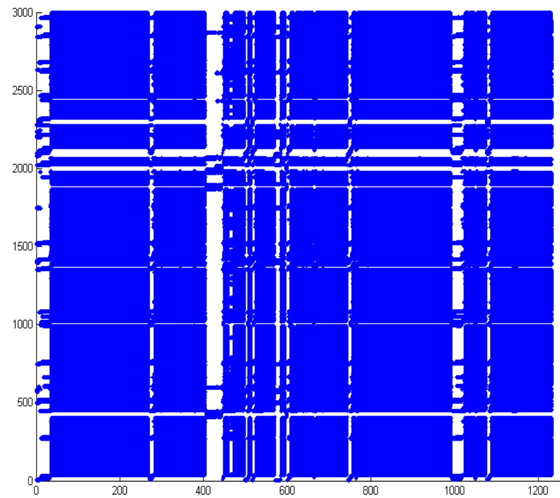
b



b



c



a

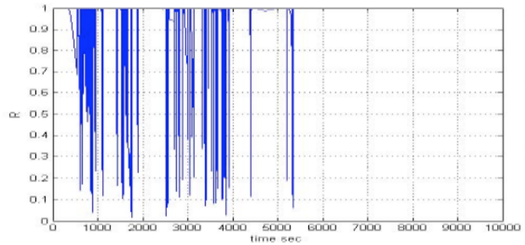
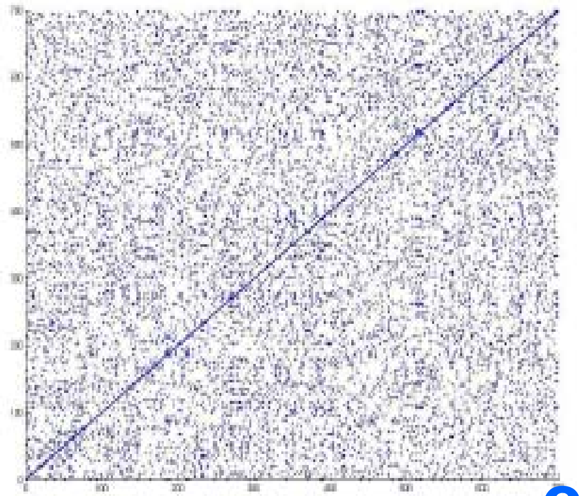


Fig. 5 Order parameter R with high coupling ($K_m = 400$).

a



c

$$q = H(\varepsilon - \| R(m) - R(h) \|)$$