CRITIS’08
13-15 October 2008
Frascati (Rome), Italy

CRITIS08

POLITECNICO DI MILANO





Using Centrality Measures to Rank the Importance of the Components of a Complex Network Infrastructure

POLITECNICO MILANO

Francesco Cadini, Enrico Zio, <u>Cristina-Andreea Petrescu</u>

Problem Statement

Safety analysis of a complex infrastructure

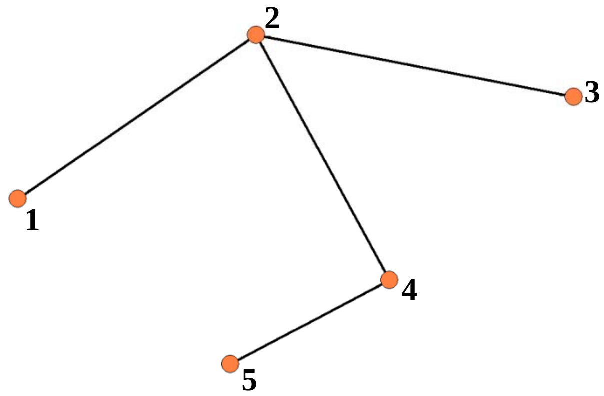
Robustness to failure?

Vulnerability to attacks?

- **Detailed Simulation**
- **PRA Tools (ET/FT)**
- **Topological Analysis**

Topological Analysis

- Connected graph $G=(N,K)$
- Adjacency matrix $\{a_{ij}\}$: $a_{ij}=1$ if there is an edge joining node i to node j and 0 otherwise
- Shortest path length matrix $\{d_{ij}\}$ from node i to node j
- Connection efficiency: $\varepsilon_{ij}=1/d_{ij}$
- Network topological efficiency: $E[G]=\frac{1}{N(N-1)}\sum_{i,j\in N, i\neq j}\varepsilon_{ij}$



$$a_{ij}=\begin{bmatrix} 0. & 1. & 0. & 0. & 0. \\ 1. & 0. & 1. & 1. & 0. \\ 0. & 1. & 0. & 0. & 0. \\ 0. & 1. & 0. & 0. & 1. \\ 0. & 0. & 0. & 1. & 0. \end{bmatrix} \quad d_{ij}=\begin{bmatrix} 0. & 1. & 2. & 2. & 3. \\ 1. & 0. & 1. & 1. & 2. \\ 2. & 1. & 0. & 2. & 3. \\ 2. & 1. & 2. & 0. & 1. \\ 3. & 2. & 3. & 1. & 0. \end{bmatrix}$$

Contribution of this Work

- Topological analysis by centrality measures, accounting for the reliability of the connections among the elements of the network infrastructure

Centrality Measures⇒Importance of Nodes

Centrality measures help highlight the nodes which:

- are most connected
- are most reliable
- are most important (their removal produce a significant drop in network functionality)
- communicate best with the others
- are most exposed to malevolent attacks

Reliability-Weighted Topological Analysis

CRITIS08

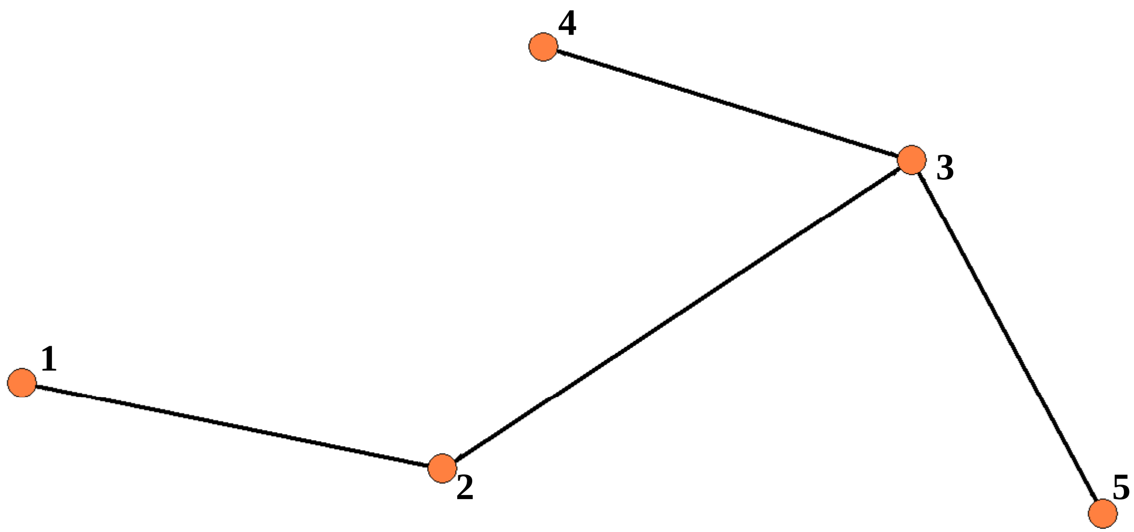
- Connection reliability p_{ij} = probability that the transmission occurs by the requirements
- Reliability matrix $\{p_{ij}\}$
- Most reliable path length from node i to node j:

$$rd_{ij} = \min_{\gamma_{ij}} \left(\frac{1}{\prod_{mn \in \gamma_{ij}} p_{mn}} \right) \qquad 1 \le rd_{ij} \le \infty$$

- Connection reliability efficiency: $r\varepsilon_{ij} = 1/rd_{ij}$
- Network reliability efficiency: $RE[G] = \frac{1}{N(N-1)} \sum_{i,j \in N, i \neq j} r\varepsilon_{ij}$

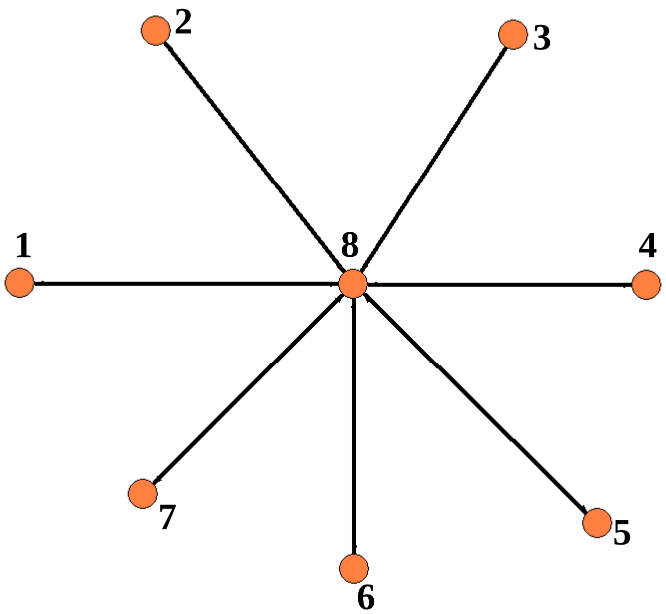
Degree Centrality Measures

Measure	Topological	Reliability-weighted topological
Degree centrality	$C_i^D = \frac{k_i}{N-1} = \frac{\sum_{j \in G} a_{ij}}{N-1}$	$RC_i^D = \frac{k_i \sum_{j \in G} p_{ij}}{(N-1)^2}$



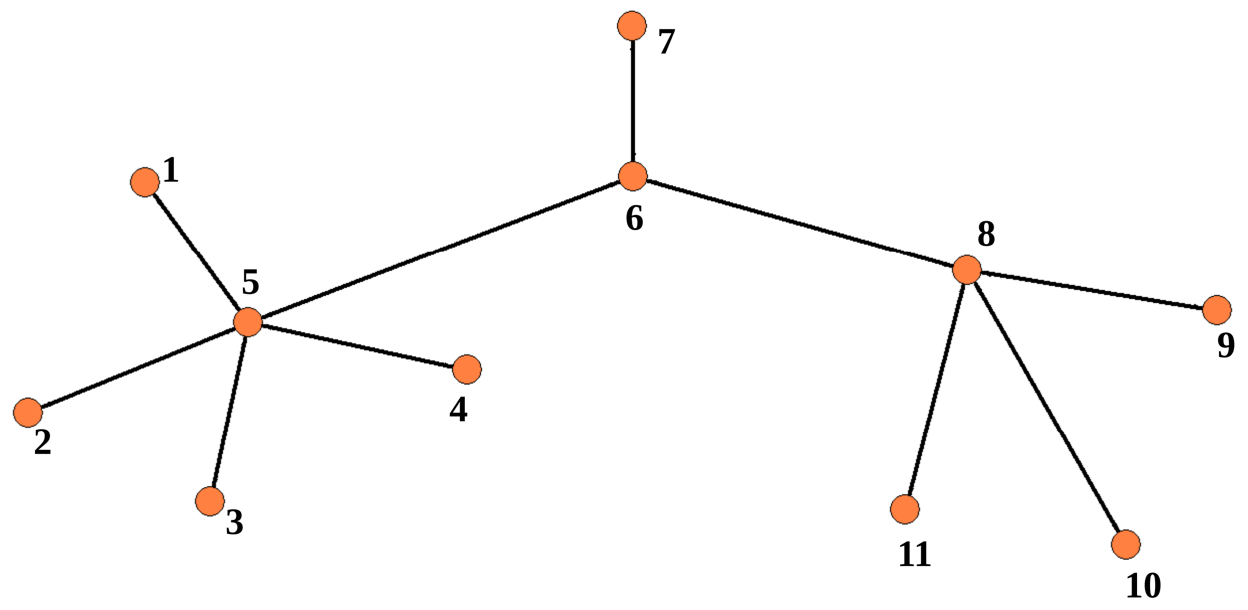
Closeness Centrality Measures

Measure	Topological	Reliability-weighted topological
Closeness centrality	$C_i^C = \frac{N-1}{\sum_{j \in G} d_{ij}}$	$RC_i^C = \frac{N-1}{\sum_{j \in G} rd_{ij}}$



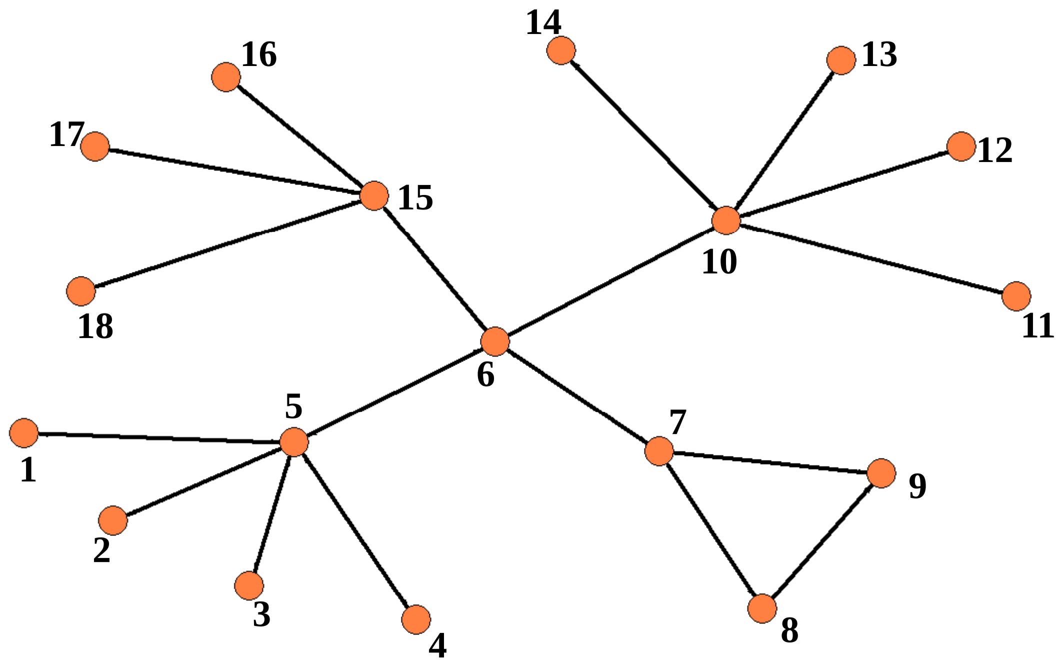
Betweenness Centrality Measures

Measure	Topological	Reliability-weighted topological
Betweenness centrality	$C_i^B = \frac{1}{(N-1)(N-2)} \sum_{j,k \in G, j \neq k \neq i} \frac{n_{jk}(i)}{n_{jk}}$	$RC_i^B = \frac{1}{(N-1)(N-2)} \sum_{j,k \in G, j \neq k \neq i} \frac{rn_{jk}(i)}{rn_{jk}}$



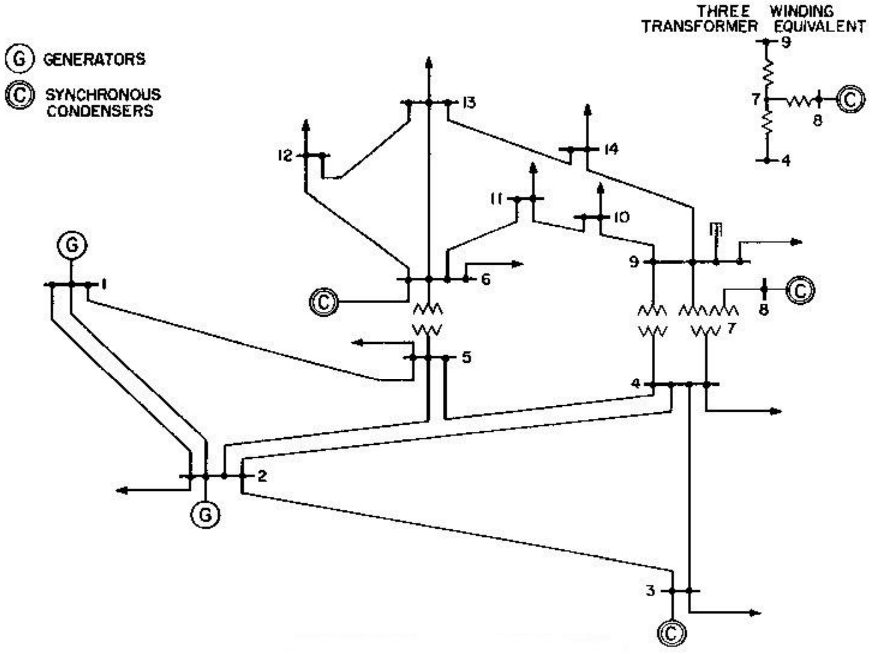
Information Centrality Measures

Measure	Topological	Reliability-weighted topological
Information centrality	$C_i^I = \frac{\Delta E}{E} = \frac{E[G]-E[G']}{E[G]}$	$RC_i^I = \frac{\Delta E_r}{E_r} = \frac{E_r[G]-E_r[G']}{E_r[G]}$

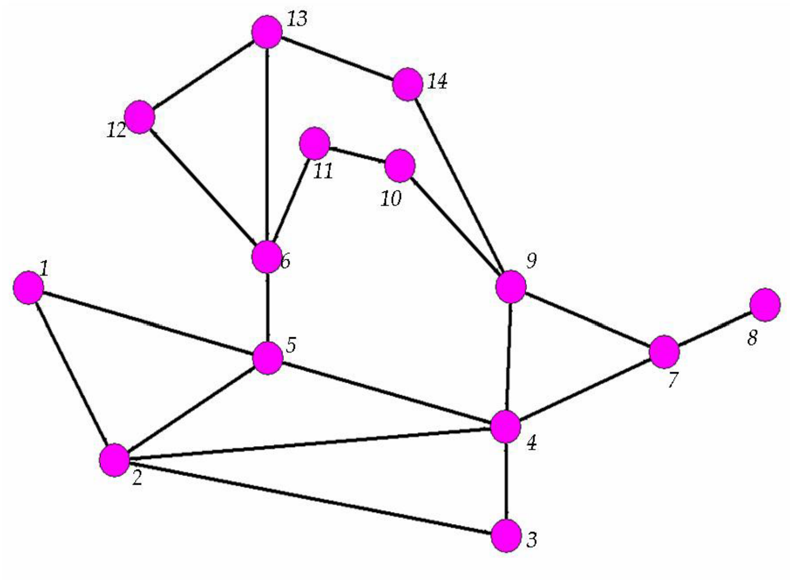


Case Study: IEEE 14 BUS

Physical system



G=(14,20)



Case Study IEEE 14 BUS: Nodes Centrality

Rank	C^D	RC^D	C^C	RC^C	C^B	RC^B	C^I	RC^I
1	4	4	4	4	4	6	4	7
2	2,5,6,9	9	5,9	9	5,9	9	7	9
3		6		5		13,14	9	4
4		7	6	7	6		6	6
5		5	2	6	7	10,11	5	5
6	7,13	2	7	14	14		2	13
7		13	14	10,13	2	4	13	14
8	1,3,10,11,12,14	10,11,12,14	10,13		13	2,7	14	10
9				11	10		10	11
10			11	2	11	5	11	8
11			3	8	1,3,8,12	1,3,8,12	3	12
12		1,3	1	12			1	2
13			12	3			12	3
14	8	8	8	1			8	1

Conclusions

- The topological concepts of centrality measures have been extended to account for the reliability of the network connections
- The reliability centrality measures have been shown capable of highlighting some network safety strengths and weaknesses otherwise not detectable on a pure topological basis
- Different centrality measures provide different insights on the network structure
- The reliability centrality measures may constitute a valuable additional tool for the network designers and managers to gain insights on the system robustness

Thank you for your attention!