A mathematical model of blackouts of an high voltage electric network

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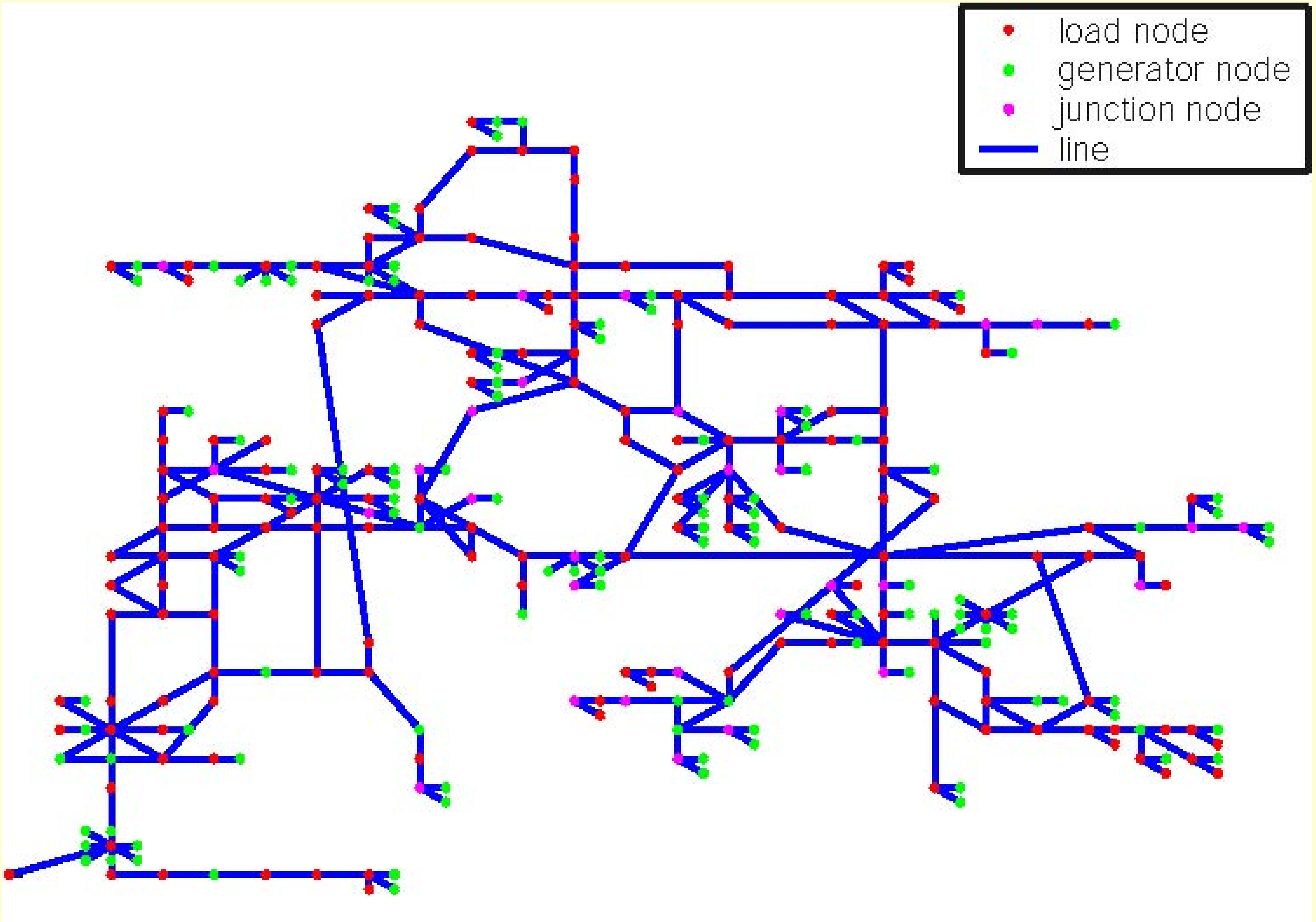
Outline of the presentation:

- Introduction
- The DC Power Flow model
- The cascading blackout model
- Numerical simulation of the cascading blackout phenomenon and the blackout size probability density function
- References

The Italian high voltage electric network (380.000 V)



A graph representing the Italian high voltage electric network

We consider the following problems:

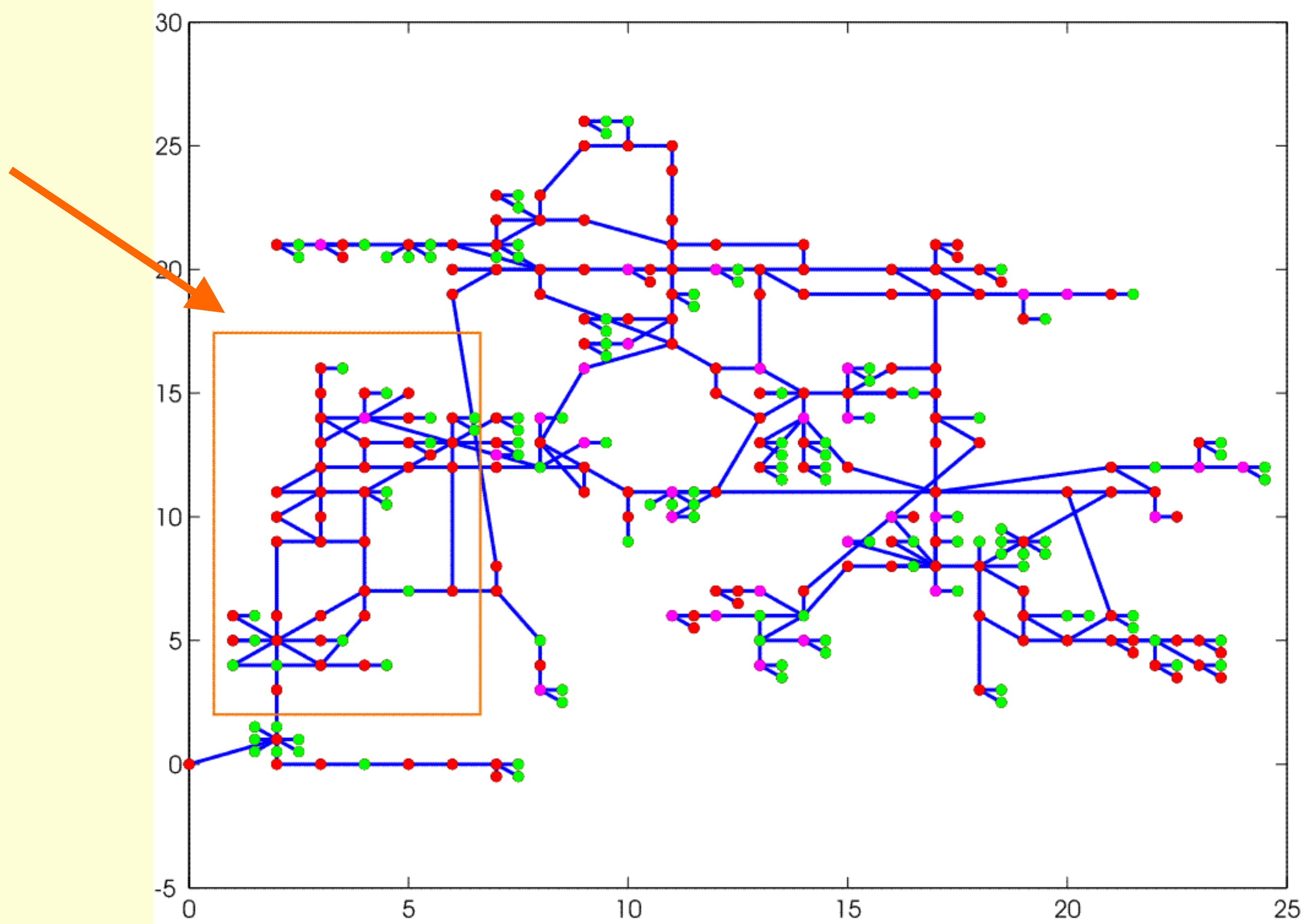
1. choice of a mathematical model describing the electric network behaviour;
2. choice of some parameters of the model in order to simulate cascading blackouts;
3. numerical simulation of cascading blackouts;
4. study of the probability density function of the random variable “blackout size” (P_S/ P_D) for increasing values of the “load” (P_D / P_C);

P_S = load power shed as effect of the blackout;
P_D = total power demand;
P_C = total generation capacity of the electric network.

5. study of the mean value of the line occupation for increasing values of the “load” (P_D/P_C);
6. repeat the analysis summarized in 1-5 when the increasing “load” (P_D / P_C) is “spatially localized”. That is we study the random variable “blackout size” (P_S / P_C) for increasing values of the “load” (P_D / P_C) when the increment of the load is localized in space.
   Typically this last type of load anomalies is caused by extreme weather conditions.

P_S = load power shed as effect of the blackout;
P_D = total power demand;
P_C = total generation capacity of the electric network.

Anomalies localized in space



The DC Power Flow model

The electric network is represented as an undirected graph made of nodes connected by branches.

The nodes can be:

1. generator nodes,
2. load nodes,
3. junction nodes.

Each branch (electric line) has its own characteristic admittance and an upper bound limit to the power that can flow along it without damages.

In the Italian high voltage electric network:

N = number of nodes = $N_G+N_L+N_J$ = 310,
N_L = number of load nodes =163,
N_G = number of generator nodes = 97,
N_J = number of junction nodes = 30,
L = number of branches = 361.

Note that there are 14 pairs of nodes connected by two branches (double lines). In our work the double lines are substituted by a single line with a suitable value of impedance, therefore we use: $L = 361 – 14 = 347$.

Notations:

For k,m = 1,2,…,N, we denote with:

- **(k,m)** the line connecting the node k to the node m,
- $\mathbf{y_{k,m}} = 1/z_{k,m} = g_{k,m} + \iota\, b_{k,m}$ the admittance of the line (k,m). Here $g_{k,m}$ and $b_{k,m}$ are, respectively, the conductance and the susceptance and ι is the imaginary unit. The quantity $z_{k,m}$ is measured in Ohms and $y_{k,m}$ is measured in Siemens. Note that there are pairs of nodes k and m that are not directly connected by a branch, in this case we put the admittance of the line (k,m) equal to zero,
- $\mathbf{P_k} = P_k^G - P_k^L$ the total real power injected into the node k. Here P_k^G and P_k^L are, respectively, the real power generated by the node k and the real power demanded by the node k. Note that P_k^G and P_k^L, k=1,2,...,N, are nonnegative real numbers, moreover we have $P_k^G = 0$ if the node k is a load node, on the contrary a generator node k may have P_k^L different from zero, in fact this non zero power may be used to keep the generator functioning. The quantity $\mathbf{P_k}$ is measured in MW (Mega Watt),

- **$F_{k,m}$** the real power flowing along the line (k,m). The quantity $F_{k,m}$ is measured in kVA (kilo Volt Ampère),
- ***P_k^G*** the upper limit of the real power that can be generated at the node k,
- ***$F_{k,m}$*** the upper power flow limit for the power flowing along the line (k,m),

- $P_D = \sum_{k=1}^{N} P_k^L$ the total power demand,

- $P_C = \sum_{k=1}^{N} P_k^G$ the total power generation capacity of the transmission network,

- $V_k = |V_k|(\cos\theta_k + \imath \sin\theta_k)$ the voltage of the node k. Here θ_k denotes the phase angle. V_k is measured in kV (kilo Volt) and θ_k is measured in radians.

We assume that the node 1 is the reference node with voltage angle normalized to 0, that is we assume $\theta_1 = 0$.

The **optimal DC Power Flow problem** consists in finding the vector $\underline{P}^G = (P_1^G, P_2^G, \ldots, P_N^G)$ and the corresponding voltage angle vector $\underline{\theta} = (\theta_1, \theta_2, \ldots, \theta_N)$ that minimize the cost function:

$$\Phi((P_1^G, P_2^G, \ldots, P_N^G)), \qquad (1)$$

subject to:

$$P_k^G - P_k^L = \sum_{m=1,\, m \neq k}^{N} F_{k,m}, \quad k = 1,2,\ldots, \quad N,$$

$$F_{k,m} = -b_{k,m}(\theta_k - \theta_m), \quad k, m = 1,2,\ldots, \quad N,$$

$$|F_{k,m}| \leq \mathcal{F}_{k,m}, \quad k,m = 1,2,\ldots,N,$$

$$0 \leq P_k^G(\underline{\theta}) \leq \mathcal{P}_k^G, \quad k = 1,2,\ldots,N,$$

$$\theta_1 = 0.$$

Note that we have $\underline{P}^G(\underline{\theta}) = A\underline{\theta} + \underline{P}^L$ where A is a NxN real matrix.

We consider two choices of the objective function of the **optimal DC Power Flow problem**:

Linear objective function:

$$\Phi((P_1^G, P_2^G, \dots, P_N^G)) = \sum_{k=1}^{N} c_k P_k^G(\underline{\theta}) - W \sum_{k=1}^{N} P_k^L, \quad (Choice\ 1)$$

Quadratic objective function:

$$\Phi((P_1^G, P_2^G, \dots, P_N^G)) = \sum_{k=1}^{N} c_k \left(P_k^G(\underline{\theta})\right)^2 - W \sum_{k=1}^{N} \left(P_k^L\right)^2 . (Choice\ 2)$$

Note that W is a positive real constant representing a cost associated to the load nodes, $\underline{\theta}=(\theta_1,\theta_2,\dots,\theta_N)$ is the voltage angle vector and that the constants $c_1,c_2,\dots,c_N$ are positive constants representing the costs of generating power at the generator nodes 1,2,…,N respectively.
We choose W=100.

We consider Choice 1 (linear objective function) when **the load anomalies are distributed on the entire transmission network**.

$$\Phi((P_1^G, P_2^G, \ldots, P_N^G)) = \sum_{k=1}^{N} c_k P_k^G(\underline{\theta}) - W \sum_{k=1}^{N} P_k^L. \quad (Choice\ 1)$$

In this case we can use the simplex algorithm to solve the optimal DC Power Flow problem (1).

We consider Choice 2 (quadratic objective function) **when the load anomalies are localized in space**.

$$\Phi((P_1^G, P_2^G, \ldots, P_N^G)) = \sum_{k=1}^{N} c_k \left(P_k^G(\underline{\theta})\right)^2 - W \sum_{k=1}^{N} \left(P_k^L\right)^2. (Choice\ 2)$$

In this case we use an interior point algorithm to solve the corresponding optimal DC Power Flow problem (1).

When the total power demand P_D grows, the solution of the DC power flow problem (1) goes through some changes. The blackout happens when the DC optimal power flow problem is unfeasible.

This is due to the violation of one or both the main constraints:

(a) $P_D \leq P_C$,

(b) $|F_{k,m}| \leq F_{k,m}$, k,m=1,2,...,N.

When (a) and/or (b) is violated it is necessary to shed loads and/or to disconnect lines to restore feasibility.

The cascading blackout model

We will consider as **blackout size measure**:

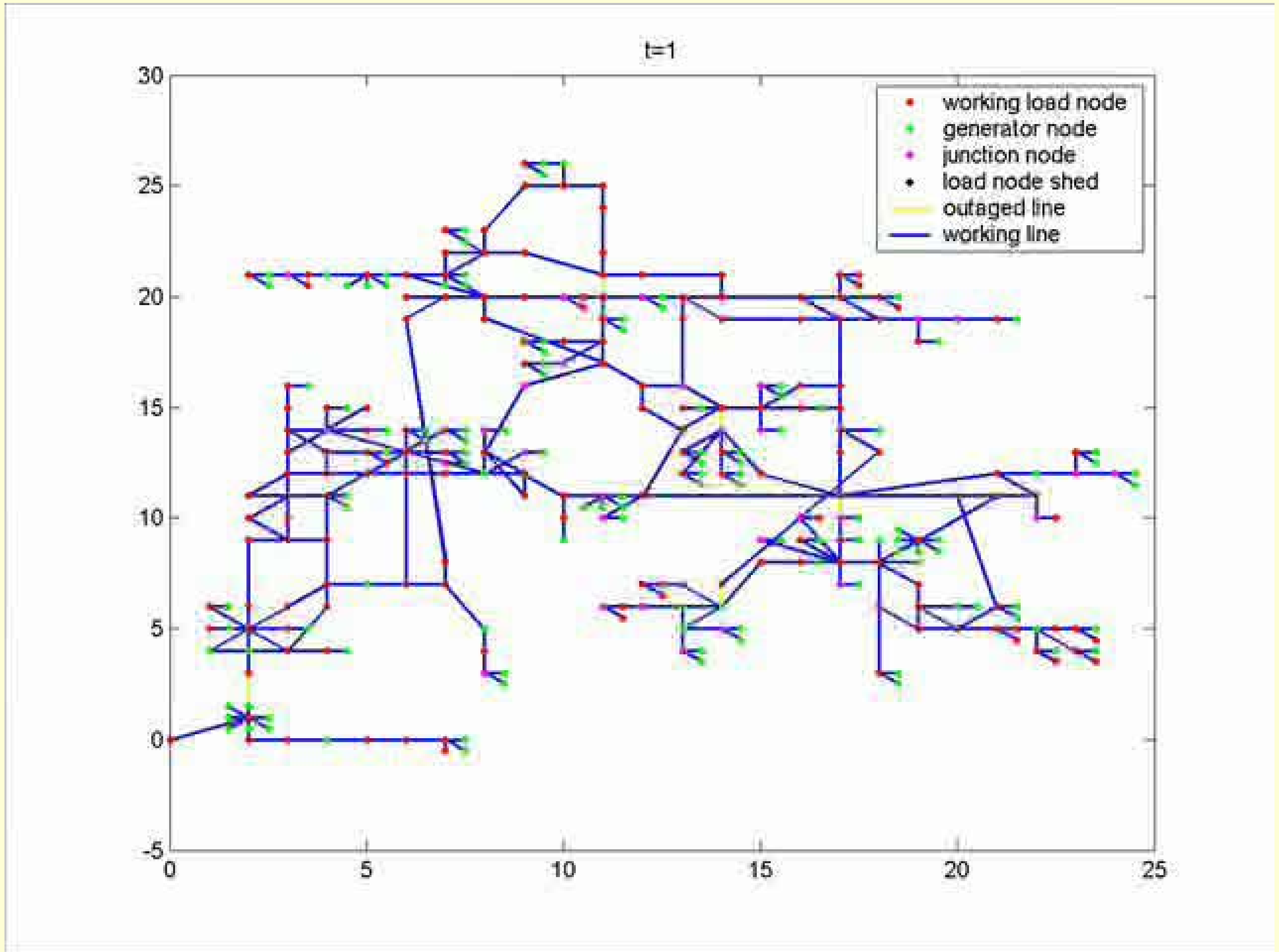
$$P_S / P_D$$

Where P_S = total power shed.

Note that $0 \leq P_S / P_D \leq 1$.

When the loads vary randomly the blackout size P_S / P_D is a random variable whose probability density function is defined implicitly by the optimal Power Flow problem (1).

In the following animation we show the cascade of events that generates a blackout for a prescribed value of P_D / P_C ($P_D / P_C = 1.0828$) . The anomalies of the load are distributed (uniformly) on the entire transmission network.



The cascading blackout numerical simulation and the blackout size probability density function

Varying randomly the loads we study the probability density function of the random variable P_S / P_D (the blackout size) for several values of P_D / P_C (mean value of the total power demand / total power capacity).The probability density function obtained from the numerical simulations is fitted with the following formulae:

- $A\, e^{-mx}$, $0 < x < 1$ **exponential law**
- B/x^{α}, $0 < x < 1$ **inverse power law**

where A,m,B and α are real constants to be determined.

The best approximations (in the least squares sense) of the probability density function of P_S / P_D are compared. We observe that:

P_D / P_C

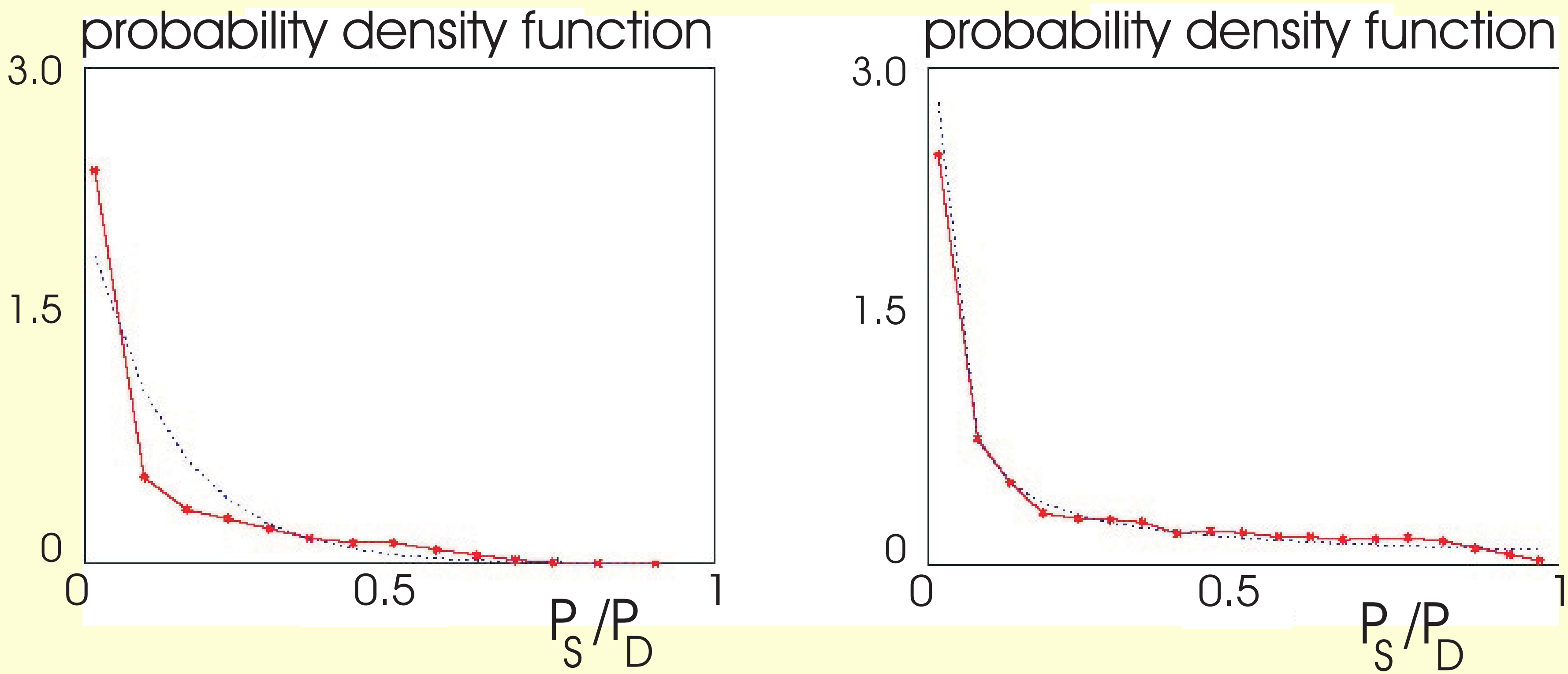
	small enough	large enough
best approximation	exponential	inverse power

The power law distribution of P_S / P_D means "large“ probability of big blackouts.

Inverse power law = fat tail of the probability density function of P_S / P_D.

Uniformly distributed anomalies

Exponential law | Power law



$P_D/P_C=0.67$ | $P_D/P_C=0.77$

The presence (or absence) of “fat tail” in the probability density function of P_S/P_D suggests the presence (or absence) of domino effect in the network.

The appearance of “fat tail" is called "phase transition".

The term "phase transition" is used to point out the analogy between the phase transition problem in statistical mechanics and the power network behaviour in critical conditions, that is in the situation where we have line overloads or the impossibility for the generators of supplying the power demanded by the load.

- Let us denote with σ_e and σ_p the mean squares errors obtained using the best fit of the pdf of P_S/P_D with the **exponential** law (σ_e) and with the **power** law (σ_p).
- We study the behavior of these two quantities σ_e and σ_p as function of the ratio total power demand over total network capacity $\mathbf{P_D/P_C}$ in two cases: uniformly distributed load anomalies on the entire network and spatially localized load anomalies.
- P_S:load power shed as effect of the blackout;
- P_D: total power demand;
- P_C: total generation capacity of the electric network.

Case 1: Blackouts due to anomalies distributed on the entire network

P_D/P_C	σ_e	σ_p
0.64	0.2409	2.3853
0.67	0.2046	1.6321
0.74	0.2540	0.5292
0.77	0.2788	0.1805
0.81	0.2442	0.0484

The transition takes place in the interval [0.74 , 0.77]

Case 2: Blackouts due to anomalies localized in space

P_D/P_C	σ_e	σ_p
0.67	0.1220	1.4362
0.69	0.3419	1.2287
0.71	0.3786	0.7206
0.75	0.4095	0.1165
0.81	0.4495	0.0656

The transition takes place in the interval [0.71 , 0.75]

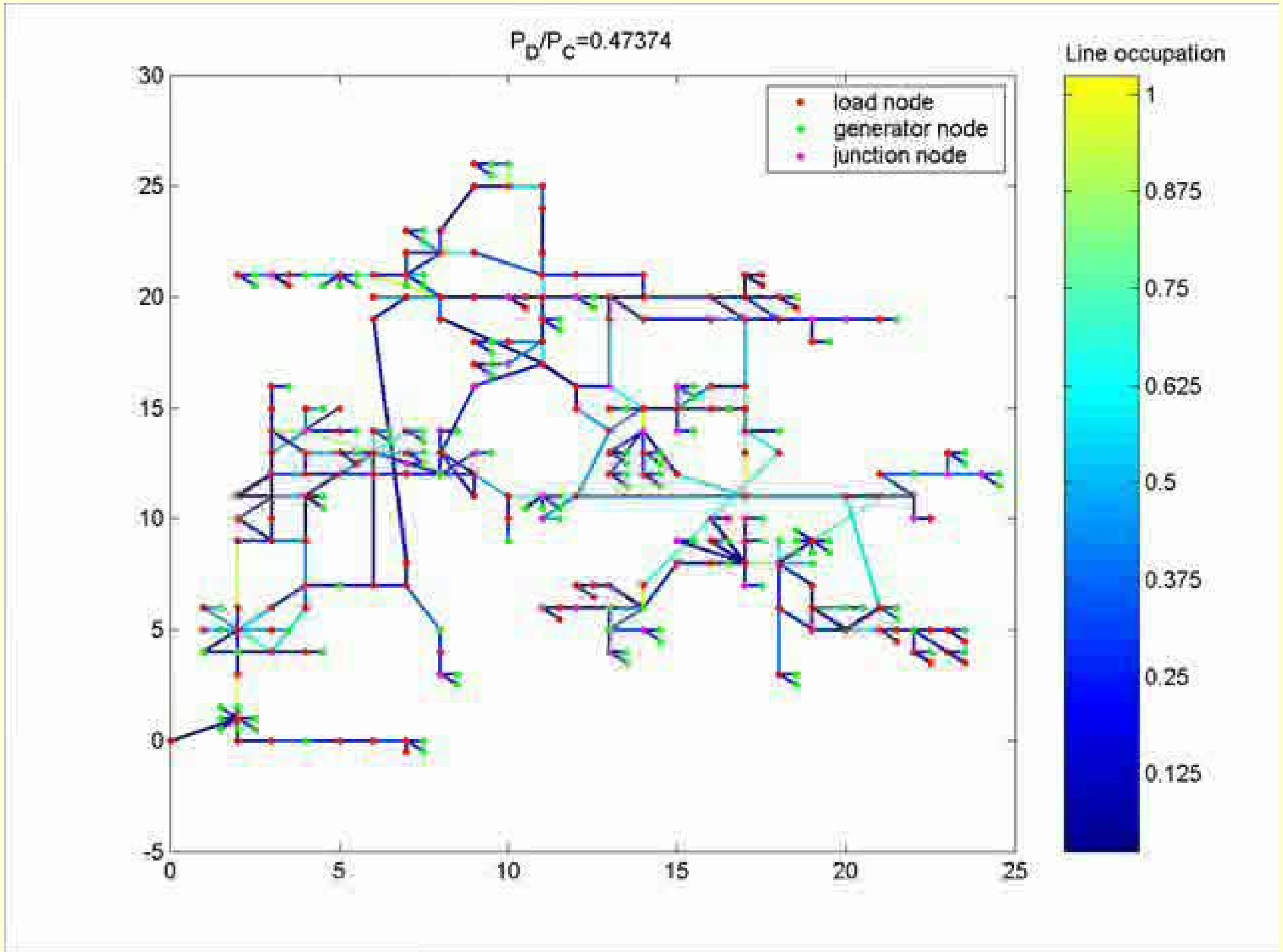
Comparison between the two kinds of blackouts

The phase transition in Case 2 (localized load anomalies) takes place in the interval [0.71, 0.75], that is “earlier” than in Case 1 (load anomalies distributed on the entire network) where it takes place in the interval [0.74,0.77]

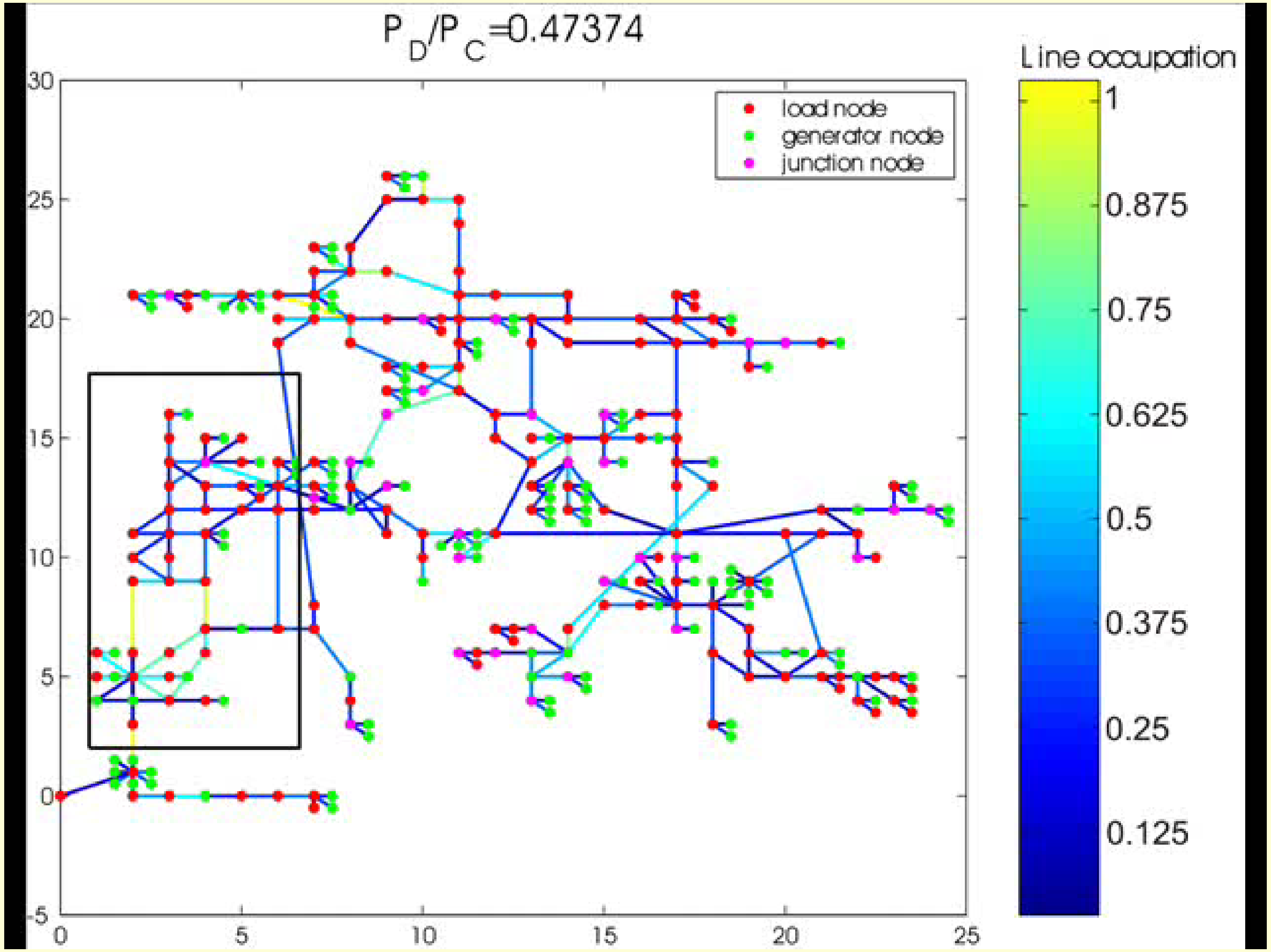
Analogies between the statistical mechanics and the network behaviour

statistical mechanics	network behaviour
temperature	total power demand (P_D)
as the water temperature approaches 100° C, there appear in the water (liquid) bubbles of water vapor (gas)	as the total power demand grows problem (1) may become unfeasible. In these cases the successive line failures and/or load sheds, operated to restore feasibility of (1) can generate bubbles of blackouts

In the following animation we show the line occupations ($|F_{k,m}| / \mathcal{F}_{k,m}$) when P_D / P_C increases (proportionally) on the entire transmission network.



In the following animation we show the line occupations ($|F_{k,m}| / F_{k,m}$) when P_D/P_C increases (proportionally) in a spatially localized zone of the transmission network.



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